

GENERALIZED QUANTUM THEORY AND BLACK HOLE EVAPORATION*

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ABSTRACT

Usually quantum theory is formulated in terms of the evolution of states through space-like surfaces. However, a generalization of this formulation is needed for field theory in spacetimes not foliable by spacelike surfaces, or in quantum gravity where geometry is not definite but a quantum variable. In particular, a generalization of usual quantum theory is needed for field theory in the spacetimes that model the process of black hole evaporation. This paper discusses a spacetime generalization of usual quantum theory that is applicable to evaporating black hole spacetimes. In this generalization, information is not lost in the process of evaporation. Rather, complete information is distributed about four-dimensional spacetime. Black hole evaporation is thus not in conflict with the principles of quantum theory when suitably generally stated.

I. PRELUDE

It is both an honor and a pleasure to participate in this celebration of Professor Humitaka Sato's 60th birthday. He is notable for his very broad range of scientific activities and the many areas of research he has pioneered. In this pioneering spirit, I offer the following essay on the connections between gravity, quantum mechanics, and black hole evaporation.

II. INTRODUCTION

The lesson of general relativity is that spacetime geometry is four dimensional and changing dynamically. Yet the familiar formulations of quantum mechanics are at variance with this lesson. They require a fixed spacetime geometry and a division of that geometry into space and time. This is because familiar quantum mechanics deals with states $|\Psi(\sigma)\rangle$ defined on spacelike surfaces, σ . The state supplies complete information about the system at any one time. It evolves unitarily between spacelike surfaces

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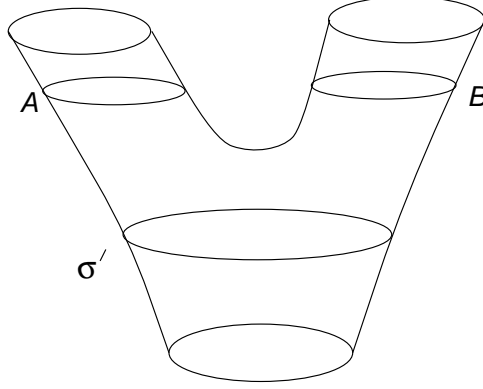


Figure 1: A spacetime with a simple change in spatial topology. There is no non-singular family of spacelike surfaces between an “initial” spacelike surface σ' and “final” spacelike surfaces A and B . The usual notion of quantum evolution must therefore be generalized to apply to spacetimes such as this.

$$|\Psi(\sigma'')\rangle = U|\Psi(\sigma')\rangle \quad (2.1)$$

or by reduction on them

$$|\Psi(\sigma)\rangle \rightarrow \frac{P|\Psi(\sigma)\rangle}{\|P|\Psi(\sigma)\rangle\|} . \quad (2.2)$$

Geometry must be fixed to define these surfaces and the spacetime must be such that it can be foliated by them. Put more technically, usual quantum mechanics requires a fixed, globally hyperbolic spacetime.

Even if the geometry is fixed, there are many spacetimes of interest for which this formulation of quantum mechanics is inadequate. Take for example, the much discussed idea of topology change illustrated in Figure 1. There is no non-singular family of spacelike surfaces interpolating between an initial surface σ' and final surfaces A and B . A generalization of familiar quantum mechanics is required for field theory in such background spacetimes. Similarly, a generalization is required for the spacetimes with closed timelike curves schematically illustrated in Figure 2 which are not foliable by any family of spacelike surfaces.

However, nowhere does the inadequacy of the usual formulations of quantum mechanics emerge more clearly than in the process of black hole evaporation. Black holes and quantum theory have been inextricably linked since Hawking’s 1974 discovery of the tunneling radiation from black holes that bears his name [1,2]. Spacetime geometries representing a process in which a black hole forms and evaporates completely have global structures summarized by the Penrose diagram in Figure 3. Let us consider for a moment the problem of quantum mechanics of matter fields in a fixed geometry with this global structure. Any pure initial state $|\Psi(\sigma')\rangle$ leads to a state of disordered Hawking radiation on a hypersurface A after the black hole has evaporated, so we have

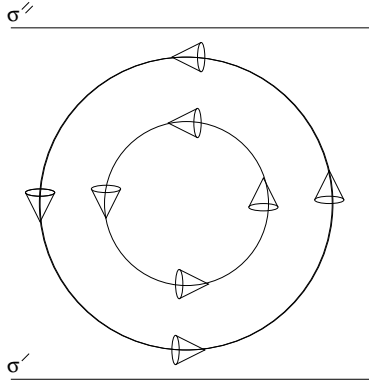


Figure 2: A spacetime with a compact region of closed timelike curves (CTC's). As a consequence of the CTC's there is no family of spacelike surfaces connecting an initial spacelike surface σ' with a final one σ'' , both outside the CTC region. The quantum evolution of a matter field therefore cannot be described by a state evolving through a foliating family of spacelike surfaces. The notion of quantum evolution must be generalized to apply to spacetimes such as this.

$$|\Psi(\sigma')\rangle\langle\Psi(\sigma')| \rightarrow \rho(A) , \quad (2.3)$$

where $\rho(A)$ is the mixed density matrix describing the disordered radiation. This transition cannot be achieved by unitary evolution. Indeed, it is difficult to conceive of *any* law for the evolution of a density matrix $\rho(\sigma)$ through a family of spacelike surfaces that would result in (2.3) because there is no non-singular family of spacelike surfaces that interpolate between σ' and A . Even classically there is no well defined notion of evolution of initial data on σ' to the surface A because of the naked singularity N . A generalization of quantum theory is needed for field theory in evaporating black hole spacetimes.

The evaporating black hole spacetimes illustrated in Figure 3 are singular. Spacetimes that are initially free from closed timelike curves but evolve them later must be singular or violate a positive energy condition [3]. Spatial topology change implies either a singularity or closed timelike curves [4]. These pathologies suggest a breakdown in a purely classical description of spacetime geometry. One might therefore hope that the difficulties with usual formulations of quantum theory in such backgrounds could be resolved in a quantum theory of gravity. But in any quantum theory of gravity spacetime geometry is not fixed. Geometry is a quantum variable — generally fluctuating and without definite value. Quantum dynamics cannot be defined by a state evolving in a given spacetime; no one spacetime is given. A generalization of usual quantum mechanics is thus needed. This need for generalization becomes even clearer if one accepts the hint from string theory that spacetime geometry is not fundamental. If spacetime is not fundamental, the fundamental law of quantum evolution cannot be the unitary progress of a state vector through surfaces *in* spacetime.

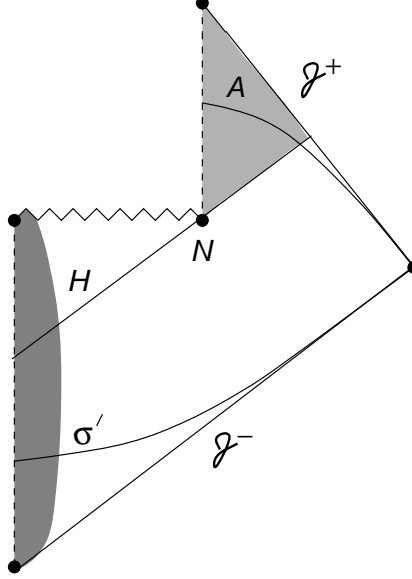


Figure 3: The Penrose diagram for an evaporating black hole spacetime. The black hole is assumed here to evaporate completely, giving rise to a naked singularity N , and leaving behind a nearly flat spacetime region (lightly shaded above). Data on surface A completely determine the evolution of fields to its future. Yet complete information about a quantum matter field moving in this spacetime is not available on A . Even if the initial state of the matter field on a surface like σ' is pure, the state of the disordered Hawking radiation on A would be represented by a density matrix. A pure state cannot evolve unitarily into a density matrix, so the usual formulation of quantum evolution in terms of states evolving through a foliating family of spacelike surfaces breaks down. The geometry of evaporating black hole spacetimes suggests why. There is no smooth family of spacelike surfaces interpolating between σ' and A and even classically there is not a well defined notion of evolution of initial data on σ' to A . The usual notion of quantum evolution must therefore be generalized to apply in spacetime geometries such as this.

The conflict between the principles of general relativity and the familiar formulation of quantum theory is usually called “the problem of time” in quantum gravity.¹ It is not a problem with giving meaning to time. Rather, it is a problem with the conflict between the preferred role of time (or spacelike surfaces) in usual quantum theory and the lesson of general relativity that there are no preferred times. A natural route forward is to generalize quantum theory so that it does not require a preferred time, or notion of spacelike surfaces, but such that the usual formulation is recovered approximately in the particular case when spacetime geometry is approximately fixed and foliable by spacelike surfaces.

A number of such generalizations have been proposed. Perhaps the oldest is the idea of Dirac, Wheeler, and others that when spacetime is dynamical, quantum mechanics should

¹For classic reviews, see Kuchař [5], Isham [6], and Unruh [7].

deal with wave functions defined, not on spacelike surfaces in a fixed spacetime (there is none), but rather on spacelike surfaces in the superspace of three-geometries. This, or spaces related to it, is the space of canonical quantum gravity. Historically, this has been an important direction for quantum gravity. However, it is still a significant departure from the lesson of general relativity that the fundamental notion is *spacetime*.

As stressed by Wald [8] the algebraic approach to quantum mechanics provides a way of dealing with field theory in non-globally hyperbolic spacetimes. Quantum dynamics is not encoded in a rule for the evolution of a state through spacelike surfaces, but rather through a spacetime algebra of local field operators. States are then defined as maps from the algebra to real numbers (*e.g.*, expected values). States referring to spacelike surfaces such as A in Figure 3 will naturally be mixed.

However, I do not want to discuss these particular proposals, but instead present a framework for treating a *whole class* of generalizations of quantum mechanics and use it to illustrate another way of making quantum theory consistent with the lessons of general relativity. The idea is to follow Feynman and put quantum mechanics in fully spacetime form; to deal with probabilities for histories rather than with alternatives confined to space-like surfaces; to deal with evolution through amplitudes for four-dimensional histories rather than by evolving states through spacelike surfaces. Such a formulation of quantum mechanics is at least general enough to use for field theory in non-globally hyperbolic spacetimes including those that describe black-hole evaporation.

To make this generalization, we will have to give up on the notion of states on a spacelike surface and quantum evolution described by the change of such state from one spacelike surface to another. This basic point is already evident classically. There is a fully four-dimensional description of any spacetime geometry in terms of four-dimensional manifold, metric, and field configurations. For globally hyperbolic geometries that four-dimensional information can be compressed into initial data on a spacelike surface, which is the classical notion of state. By writing the Einstein equation in $3 + 1$ form, the four dimensional description can be recovered by evolving the state through a family of spacelike surfaces. However such compression is not possible in spacetimes like the evaporating black hole spacetime illustrated in Figure 3. Only a four-dimensional description is possible.

Analogously, there is a fully four-dimensional formulation of quantum field theory in background spacetime geometries in terms of Feynman's sum-over-field-histories. Transition amplitudes between spacelike surfaces are specified directly from the four-dimensional action S by sums over field histories of the form

$$\sum_{\text{histories}} \exp[iS(\text{field history})/\hbar] . \quad (2.4)$$

When the background geometry is globally hyperbolic, these transition amplitudes between spacelike surfaces can be equivalently calculated by evolving a quantum state through an interpolating family of spacelike surfaces. However, if the geometry is not globally hyperbolic, we cannot expect such a $3 + 1$ formulation of quantum dynamics any more than we can for the classical theory. Following Feynman, however, we expect a fully four-dimensional spacetime formulation of quantum theory to supply the necessary generalization applicable to field theory in evaporating black hole spacetimes and to the other examples we have mentioned. We shall describe this generalization and its consequences in this article. Think

four-dimensionally. When one does there is no problem of time and no necessary conflict between quantum mechanics and black hole evaporation.

III. GENERALIZED QUANTUM THEORY.

Quantum mechanical interference between histories is an obstacle to thinking four dimensionally. Most generally, we would like to predict probabilities for the individual members of sets of alternative four-dimensional histories of a closed system. But interference prevents the consistent assignment of probabilities to many sets of histories. This is clearly illustrated in the famous two-slit experiment shown in Figure 4. There are two possible histories for an electron which proceeds from the source to a point y at the detecting screen, defined by which of the two slits (A or B) it passes through. It is not possible to assign probabilities to this set of two histories. It would be inconsistent to do so because the probability to arrive at y is not the sum of the probabilities to arrive at y *via* the two possible histories:

$$p(y) \neq p_A(y) + p_B(y) . \quad (3.1)$$

That is because in quantum mechanics probabilities are squares of amplitudes and

$$|\psi_A(y) + \psi_B(y)|^2 \neq |\psi_A(y)|^2 + |\psi_B(y)|^2 \quad (3.2)$$

In a quantum theory a rule is therefore needed to specify which sets of alternative, four-dimensional histories may be assigned probabilities and which may not. The rule in usual quantum mechanics is that probabilities can be assigned to the histories of the outcomes of *measurements* and not in general otherwise. Interference between histories is destroyed by the measurement process, and probabilities may be consistently assigned. However, this rule is too special to apply in the most general situations and certainly insufficiently general for cosmology. Measurements and observers, for example, were not present in the early universe when we would like to assign probabilities to histories of density fluctuations in matter fields or to the evolution of spacetime geometry.

The quantum mechanics of closed systems² relies on a more general rule whose essential idea is easily stated: A closed system is in some initial quantum state $|\Psi\rangle$. Probabilities can be assigned to just those sets of histories for which there is vanishing interference between the individual members as a consequence of the state $|\Psi\rangle$ the system is in. Such sets of histories are said to *decohere*. Histories of measurements decohere as a consequence of the interaction between the apparatus and measured subsystem. Decoherence thus contains the rule of usual quantum mechanics as a special case. But decoherence is more general. It permits assignment of probabilities to alternative orbits of the moon or alternative histories of density fluctuations in the early universe when the initial state is such that these alternatives decohere whether or not the moon or the density fluctuations are receiving the attention of observers making measurements.

²See, *e.g.* Omnès [9] for a review.

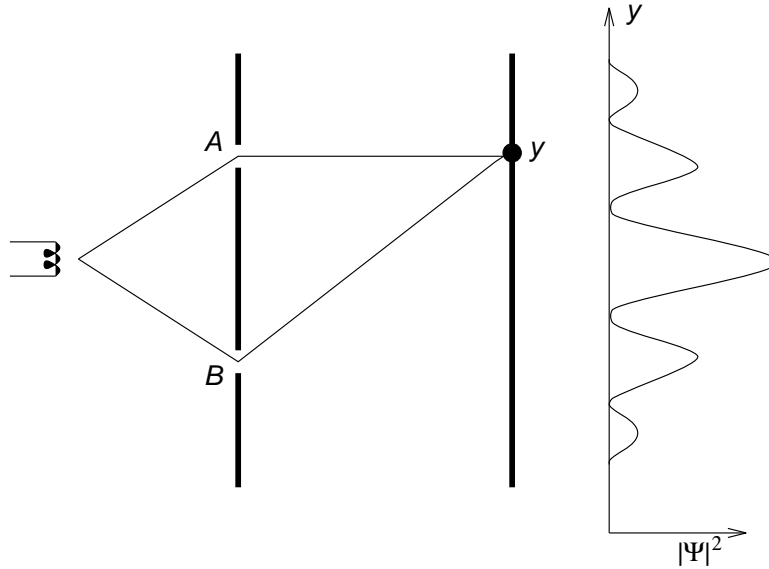


Figure 4: *The Two-Slit Experiment.* An electron gun at left emits an electron traveling towards a screen with two slits and then on to detection at a further screen. Two histories are possible for electrons detected at a particular point on the right-hand screen defined by whether they went through slit A or slit B. Probabilities cannot be consistently assigned to this set of two alternative histories because they interfere quantum mechanically.

The central element in a quantum theory based on this rule is the measure of interference between the individual coarse-grained histories c_α , $\alpha = 1, 2, \dots$ in a set of alternative histories. This measure is called the *decoherence functional*, $D(\alpha', \alpha)$. A set of coarse-grained histories decoheres when $D(\alpha', \alpha) \approx 0$ for all pairs (α', α) of distinct histories. We have generally specified a generalized quantum theory when we have specified:

- (i) The sets of alternative fine-grained histories of the system. These are the most refined descriptions of the system possible. The world line of a particle or a four-dimensional field configuration are examples.
- (ii) The allowed coarse grainings of these fine-grained histories, generally partitions of a fine-grained set into mutually exclusive classes called coarse-grained histories. The class of world lines crossing a particular region of space at a definite time is an example of a coarse-grained history for a particle.
- (iii) The decoherence functional measuring the interference between the individual members of a set of coarse-grained histories. This should satisfy a number of general properties such as the principle of superposition which Isham [10] has characterized axiomatically. Usual quantum mechanics is one way of implementing these properties, but not the only way. Therein lie the possibilities for its generalization.

IV. SPACETIME GENERALIZED QUANTUM MECHANICS

Feynman's sum-over-histories ideas may be used with the concepts of generalized quantum theory to construct a fully four-dimensional formulation of the quantum mechanics of a matter field $\phi(x)$ in a fixed background spacetime. (See, *e.g.* Hartle [11].) We shall sketch this spacetime quantum mechanics in what follows. We take the dynamics of the field to be summarized by an action $S[\phi(x)]$ and denote the initial state of the closed field system by $|\Psi\rangle$.

The set of fine-grained histories are the alternative, four-dimensional configurations of the field on the spacetime. Sets of alternative coarse-grained histories to which the theory assigns probabilities if decoherent are partitions of these field configurations into exclusive classes $\{c_\alpha\}$, $\alpha = 1, 2, \dots$. For example, the alternative that the field configuration on a spacelike surface σ has the value $\chi(\mathbf{x})$ corresponds to the class of four-dimensional field configurations which take this value on σ . In flat spacetime the probability of this alternative is the probability for the initial state $|\Psi\rangle$ to evolve to a state $|\chi(\mathbf{x}), \sigma\rangle$ of definite field on σ . The history where the field takes the value $\chi'(\mathbf{x})$ on surface σ' and $\chi''(\mathbf{x})$ on a later surface σ'' corresponds to the class of four-dimensional field configurations which take these values on the respective surfaces, and so on.

The examples we have just given correspond to the usual quantum mechanical notion of alternatives at a definite moment of time or a sequence of such moments. However, more general partitions of four-dimensional field configurations are possible which are not at any definite moment of time or series of such moments. For example, the four-dimensional field configurations could be partitioned by ranges of values of their averages over a region extending over both space and time. Partition of four-dimensional histories into exclusive classes is thus a fully four-dimensional notion of alternative for quantum mechanics.

Branch state vectors corresponding to individual classes c_α in a partition of the fine-grained field configurations $\phi(x)$ can be constructed from the sum over fields in the class c_α . We write schematically

$$C_\alpha |\Psi\rangle = \int_{c_\alpha} \delta\phi \exp(iS[\phi(x)]/\hbar) |\Psi\rangle. \quad (4.1)$$

It is fair to say that the definition of such integrals has been little studied in interesting background spacetimes, but we proceed assuming a careful definition can be given even in singular spacetimes such as those discussed earlier. Even then some discussion is needed to explain what (4.1) means as a formal expression. In a globally hyperbolic spacetime we can define an operator C_α corresponding to the class of histories c_α by specifying the matrix elements

$$\langle \chi''(\mathbf{x}), \sigma'' | C_\alpha | \chi'(\mathbf{x}), \sigma' \rangle = \int_{[\chi' c_\alpha \chi'']} \delta\phi \exp(iS[\phi(x)]/\hbar). \quad (4.2)$$

The sum is over all fields in the class c_α that match $\chi'(\mathbf{x})$ and $\chi''(\mathbf{x})$ on the surfaces σ' and σ'' respectively. This operator can act on $|\Psi\rangle$ by taking the inner product with its field representative $\langle \chi'(\mathbf{x}), \sigma' | \Psi \rangle$ on a spacelike surface far in the past. By pushing σ'' forward to late times we arrive at the definition of $C_\alpha |\Psi\rangle$. The same procedure could be used to define branch state vectors in spacetimes with closed timelike curves (Figure 2), in spacetimes with spatial topology change (Figure 1), and in evaporating black hole spacetimes (Figure 3). The

only novelty in the latter two cases is that $C_\alpha|\Psi\rangle$ lives on the product of two Hilbert spaces. There are the Hilbert spaces on the two legs of the trousers in the spatial topology change case and, in the black hole case, there are the Hilbert space of states inside the horizon and the Hilbert space of states on late time surfaces after the black hole has evaporated.

The decoherence functional is then

$$D(\alpha', \alpha) = \mathcal{N} \langle \Psi | C_\alpha^\dagger C_{\alpha'} | \Psi \rangle , \quad (4.3)$$

where \mathcal{N} is a constant to ensure the normalization condition

$$\Sigma_{\alpha\alpha'} D(\alpha', \alpha) = 1 . \quad (4.4)$$

A set of alternative histories decoheres when the off-diagonal elements of $D(\alpha', \alpha)$ are negligible. The probabilities of the individual histories are

$$p(\alpha) = D(\alpha, \alpha) = \mathcal{N} \|C_\alpha|\Psi\rangle\|^2 . \quad (4.5)$$

There is no issue of “conservation of probability” for these $p(\alpha)$; they are not defined in terms of an evolving state vector. As a consequence of decoherence, the $p(\alpha)$ defined by (4.5) obey the most general probability sum rules including, for instance, the elementary normalization condition $\Sigma_\alpha p(\alpha) = 1$ which follows from (4.4).

	Usual Quantum Mechanics	Generalized Quantum Mechanics
Dynamics	$e^{-iHt} \Psi\rangle$ $P \Psi\rangle/\ P \Psi\rangle\ $	$\sum_{\substack{\text{histories} \\ \in c_\alpha}} e^{iS(\text{history})/\hbar} \Psi\rangle$
Alternatives	On spacelike surfaces or sequences of surfaces	Arbitrary partitions of fine-grained histories
Probabilities assigned to	Histories of measurement outcomes	Decohering sets of histories

This spacetime generalized quantum theory is only a modest generalization of usual quantum mechanics in globally hyperbolic backgrounds as the above table shows. The two laws of evolution (2.1) and (2.2) have been unified in a single sum-over-histories expression. The alternatives potentially assigned probabilities have been generalized to include ones that extend in time and are not simply the outcomes of a measurement process. These generalizations put the theory in fully four-dimensional form.

When spacetime is fixed and foliable by spacelike surfaces, this spacetime formulation of quantum theory reduces to the familiar formulation in terms of states on spacelike surfaces. Pick any spacelike surface σ and define, as in (4.1),

$$\Psi[\chi(\mathbf{x}), \sigma] = \int_{[\Psi, \chi(\mathbf{x})]} \delta\phi \exp(i S[\phi(x)]) |\Psi\rangle \quad (4.6)$$

where the integral is over field configurations to the past of σ that match the given spatial configuration $\chi(\mathbf{x})$ on σ . As σ is pushed forward in time, the resulting state vector evolves

unitarily. However, it cannot be simply pushed through a topology change, a region of closed timelike curves, or the naked singularity in an evaporating black hole spacetime. There the description of evolution of states breaks down.

Thus, for coarse grainings that are defined by alternatives on spacelike surfaces, the probabilities predicted by this path integral formulation coincide with those predicted by evolving states in globally hyperbolic spacetimes. But, given appropriate definitions of the path integral, they predict probabilities for alternatives in spacetimes with topology change, spacetimes with closed spacelike curves, and evaporating black hole spacetimes.

Even when spacetime is not fixed but fluctuating quantum mechanically, we can formulate a quantum mechanics of spacetime geometry within generalized quantum theory. The histories are histories of four-dimensional geometry. The coarse grainings are partitions of these into diffeomorphism invariant classes, and a decoherence functional can be constructed along path integral lines. So formulated there is no problem with time in quantum gravity. Of course, *M*-theory suggests that spacetime itself is not a fundamental notion. But even there we may expect to find notions of histories, coarse grainings, and a theory of generalized quantum gravity free from the problem of time.

The process of black hole evaporation is thus not in conflict with the principles of quantum mechanics suitably generally stated. It is not in conflict with quantum evolution described four-dimensionally. It is only in conflict with the narrow idea that this evolution be reproduced by evolution of a state vector through a family of spacelike surfaces.

V. INFORMATION

A spacetime formulation of quantum mechanics requires a spacetime notion of information that is also in fully four-dimensional form. In this section we describe a notion of the information available in *histories* and not just in alternatives on a single spacelike surface.³ We then apply this to discuss the question of whether information is lost in an evaporating black-hole spacetime.

In quantum mechanics, a statistical distribution of states is described by a density matrix. For the forthcoming discussion it is therefore necessary to generalize the previous considerations a bit and treat mixed density matrices ρ as initial conditions for the closed system as well as pure states $|\Psi\rangle$. To do this it is only necessary to replace (4.3) with

$$D(\alpha', \alpha) = \text{Tr} \left(C_{\alpha'} \rho C_{\alpha}^{\dagger} \right) . \quad (5.1)$$

A generalization of the standard Jaynes construction (See, *e.g.* Jaynes [14] gives a natural definition of the missing information in a set of histories $\{c_{\alpha}\}$. We begin by defining the entropy functional on density matrices:

$$\mathcal{S}(\tilde{\rho}) \equiv -\text{Tr} (\tilde{\rho} \log \tilde{\rho}) . \quad (5.2)$$

³ The particular construction we use is due to Gell-Mann and Hartle [12]. There are a number of other ideas, *e.g.* Isham and Linden [13] with which the same points about information in an evaporating black-hole spacetime could be made.

With this we define the *missing information* $S(\{c_\alpha\})$ in a set of histories $\{c_\alpha\}$ as the maximum of $\mathcal{S}(\tilde{\rho})$ over all density matrices $\tilde{\rho}$ that preserve the predictions of the true density matrix ρ for the decoherence and probabilities of the set of histories $\{c_\alpha\}$. Put differently, we maximize $\mathcal{S}(\tilde{\rho})$ over $\tilde{\rho}$ that preserve the decoherence functional of ρ defined in terms of the corresponding class operators $\{C_\alpha\}$. Thus, the missing information in a set of histories $\{c_\alpha\}$ is given explicitly by:

$$S(\{c_\alpha\}) = \max_{\tilde{\rho}} \left[\mathcal{S}(\tilde{\rho}) \right]_{\{Tr(C_{\alpha'} \tilde{\rho} C_\alpha^\dagger) = Tr(C_{\alpha'} \rho C_\alpha^\dagger)\}} . \quad (5.3)$$

Complete information, S_{compl} — the most one can have about the initial ρ — is found in the decoherent set of histories with the least missing information.

$$S_{\text{compl}} = \min_{\substack{\text{decoherent} \\ \{c_\alpha\}}} \left[S(\{c_\alpha\}) \right] . \quad (5.4)$$

In usual quantum mechanics it is not difficult to show that S_{compl} defined in this way is exactly the missing information in the initial density matrix ρ

$$S_{\text{compl}} = \mathcal{S}(\rho) = -Tr(\rho \log \rho) . \quad (5.5)$$

Information in a set of histories and complete information are *spacetime* notions of information whose construction makes no reference to states or alternatives on a spacelike surfaces. Rather the constructions are four-dimensional making use of histories. Thus for example, with these notions one can capture the idea of information in entanglements in time as well as information in entanglements in space.

One can find *where* information is located in spacetime by asking what information is available from alternatives restricted to fields in various spacetime regions. For example, alternative values of a field average over a region R refer only to fields inside R . The missing information in a region R is

$$S(R) = \min_{\substack{\text{decohering } \{c_\alpha\} \\ \text{referring to } R}} S(\{c_\alpha\}) . \quad (5.6)$$

When R is extended over the whole of the spacetime the missing information is complete. But it is an interesting question what *smaller* regions R contain complete information, $S(R) = S_{\text{compl}}$.

When spacetime is globally hyperbolic and quantum dynamics can be described by states unitarily evolving through a foliating family of spacelike surfaces, it is a consequence of the definitions (5.5) and (5.6) that complete information is available on each and every spacelike surface σ

$$S(\sigma) = S_{\text{compl}} = -Tr(\rho \log \rho) . \quad (5.7)$$

However, in more general cases, only incomplete information may be available on any space-like surface, and complete information may be distributed about the spacetime. The evaporating black hole spacetimes in Figure 5 are an example.

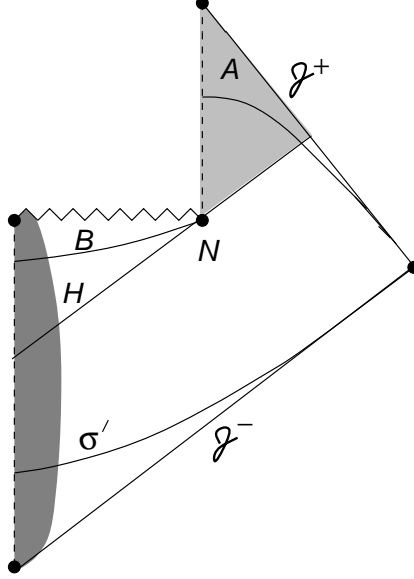


Figure 5: Quantum evolution in an evaporating black hole spacetime can be described four-dimensionally using a Feynman sum-over-histories. However, that evolution is not expressible in $3 + 1$ terms by the smooth evolution of a state through spacelike surfaces.

Complete information is not recoverable on surface A because of correlations (entanglements) between the field on A and the field on a surface like B . Even though complete information is not necessarily available on any one spacelike surface, it is not lost in an evaporating black hole geometry. It is distributed over four-dimensional spacetime.

There is no reason to expect to recover complete information on a surface like A in Figure 5. One should rather expect complete information to be available in the spacetime region which is the union of A and a surface like B . Even though this region is not a spacelike surface there are still entanglements “in time” between alternatives on A and alternatives on B that must be considered to completely account for missing information.

The situation is not so very different from that of the “trousers” spacetime sketched in Figure 1. There, complete information about an initial state on σ' is plausibly not available on surfaces A or B separately. That is because there will generally be correlations (entanglements) between alternatives on A and alternatives on B . Complete information is thus available in spacetime even if not available on any one spacelike surface like A . The surfaces A and B of this example are similar in this respect to the surfaces A and B of the evaporating black hole spacetime in Figure 5.

Thus even though it is not completely available on every spacelike surface like A , information is not lost in evaporating black-hole spacetimes. Complete information is distributed about the spacetime.

Bob Wald, who I think agrees with most of these ideas, thinks this is a bad way of characterizing the situation. He says that to say that information is not lost, is like saying “I can’t find my keys which I had yesterday, but they are not lost, they can be found in the past in spacetime”. Information depends on coarse graining. Typically, information *is* incomplete

—“lost” — in any given set of coarse-grained alternatives such as those characterizing our memories or those available to observers at infinity in an evaporating black hole spacetime. Here, when we say information “is not lost”, we mean that there is some coarse-grained set of alternatives by which complete information might have been recovered in principle. That is the case in black hole evaporation.

VI. CONCLUSION

String theory may be a quantum theory of gravity in which there is unitary evolution between pre-collapse and the evaporated states of a black hole. Whether or not this is true, some generalization of usual quantum mechanics will be necessary when spacetime geometry is not fixed. Further, we have sketched the principles of a spacetime generalized quantum theory applicable both to fixed spacetime models of black hole evaporation and more generally to quantum gravity. Thus, black-hole evaporation is not in conflict with the principles of quantum theory when suitably generally stated. Whether or not the evaporation process can be described by a unitary S -matrix, generalized quantum theory is ready to describe it.

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